Extraction of useful information content from nonstationary signals

2nd Conference on Machine Learning for Gravitational Waves, Geophysics and Control Systems

Assist. Prof. Nicoletta Saulig, PhD
Jurja Dobrile University of Pula

Content

• Introduction

• Time-frequency distributions

• Methods for the extraction of useful information content
  ➢ LRE method
  ➢ ICI rule based method

• Applications

• Conclusion
Nonstationary signals

- Nonstationary signals are characterized by variable frequency content, often multiple components, and noise.
- Classical signal representations are often not exhaustive in representing such signals.
Time-frequency distributions

- Two variable functions $\rho(n, m)$ showing how the frequency content of the signal changes over time
- The Wigner-Ville distribution

$$\rho_{WV}(t, f) = \mathcal{F}_{\tau \rightarrow f} \left\{ z \left( t + \frac{\tau}{2} \right) z^* \left( t - \frac{\tau}{2} \right) \right\}$$ (1)
Time-frequency distributions

- The Quadratic class of TFDs
  \[ \rho(t, f) = F_{\tau \rightarrow f} \{ G(t, \tau)^* \left[ z \left( t + \frac{\tau}{2} \right) z^* \left( t - \frac{\tau}{2} \right) \right] \}, \quad (2) \]
  (Discrete form \( \rho(n, m) = F_{l \rightarrow m} \{ G(n, l)^* \left[ z(n + l) z^*(n - l) \right] \} )

- Different signal components are separate energy clusters in the TF plane

\[ \text{WV TFD} \]

\[ \text{SP TFD} \]
TFDs and useful information

- Real-life signals are often collected in noisy environments, corrupting effects of which are difficult to suppress during the stage of signals acquisition and transmission.

- Signal components are concentrated energy regions emerging from the background noise which is sparse over the TF plane.
TFDs’ structure

• I) If the analyzed signal in discrete-time has dimension N, and its TFD is computed over M frequency bins, such that TFD dimension is $N \times M$, then it is assumed that the number of components $J$ is significantly smaller than the dimension of the signal and the signal components are distributed in the plane in a sparse way, such that to the total TF support of the signal components $E$ will apply $E \ll N \times M$

• II) Signal components are concentrated and continuous energy regions, emerging from the tendentially flat noise, describing their respective instantaneous frequencies, while cross-terms minimization is assumed
Amplitude segmentation

- An amplitude segmentation of data in a TFD can be performed by the K-means algorithm.

- Considering the set of observations $\mathcal{C} = \{\rho(n, m)\}$ the K-means algorithm partitions these observations into K subsets $C_k$ in order to minimize the within-cluster sum of squares with the minimal sum

$$\arg\min_{C} \sum_{k=1}^{K} \sum_{\rho(n,m) \in C_k} \|\rho(n,m) - \mu_k\|^2$$

where $\mu_k$ is the mean of each set $C_k$.

- The partitioning of the TFD into K classes itself gives no information on which classes contain the useful information and which classes contain noise coefficients.
K-means segmentation
Classes discrimination - LRE

- Noise classes - large TF supports
- Useful information content classes - small TF supports

- The structural difference between the noise and signal components classes can be achieved by the local Rényi entropy

\[ H_k(p) = \frac{1}{1-\alpha} \log_2 \sum_n \sum_m \left( \frac{\rho(n,m)}{\Sigma_n \Sigma_m \rho(n,m)} \right)^\alpha, \quad p - \frac{\Delta n}{2} \leq n < p + \frac{\Delta n}{2} \]  \hspace{1cm} (4)

that estimates the frequency supports of energy regions of the different classes
LRE estimation
Classes discrimination-LRE

- The algorithm assumes that the LRE of the first class $H_1(p)$, containing the elements with smallest amplitudes, can be classified as noise, and that it can be discarded.
- Thus, all the classes that for at least one instant $p$ satisfy $H_k(p) \geq H_1(p)$, are also considered as noise.
- Useful information: starting from the two of the remaining classes that exhibit the closest entropy functions.
Useful information extraction with reduced computational complexity
Classes discrimination - ICI

- The algorithm looks for the overlapping of the largest lower interval and the smallest upper interval and all the previous intervals as long as it exists.
Results and conclusion

- Successful tests on real life signals and different TFDs
Many thanks
Future work - band limited approach
Future work - band limited approach

- **White noise** - constant power spectral densities
- Unlike white noise, which presents spectrum with equal power within any equal interval of frequencies, colored noise exhibits different spectral profiles

\[ S(f) \propto \frac{1}{f^\alpha} \quad (4) \]
### Signal 1

<table>
<thead>
<tr>
<th>SNR</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>K-m/LRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 dB</td>
<td>18.42</td>
<td>11.19</td>
<td>8.57</td>
<td>8.29</td>
<td>8.65</td>
<td>8.26</td>
</tr>
<tr>
<td>-3 dB</td>
<td>21.69</td>
<td>15.56</td>
<td>12.31</td>
<td>11.11</td>
<td>10.78</td>
<td>10.69</td>
</tr>
</tbody>
</table>

### Signal 2

<table>
<thead>
<tr>
<th>SNR</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>K-m/LRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 dB</td>
<td>15.30</td>
<td>10.39</td>
<td>10.52</td>
<td>11.51</td>
<td>12.02</td>
<td>10.35</td>
</tr>
<tr>
<td>-3 dB</td>
<td>17.59</td>
<td>11.56</td>
<td>11.56</td>
<td>11.92</td>
<td>12.54</td>
<td>11.38</td>
</tr>
</tbody>
</table>
Results and conclusion

- Successful tests on real life signals

- Encouraging results for implementation in signal processing applications
Thank you!
\[ DB = \frac{1}{n_c} \sum_{i=1}^{n_c} R_i, \]

\[ R_i = \max_{j=1,...,n_c, i \neq j} (R_{ij}), \quad i=1,...,n_c \]

\[ R_i = \frac{s_i + s_j}{d_{ij}} \]

\[ d_{ij} = d(v_i,v_j) \]

\[ s_i = \frac{1}{\|c_i\|} \sum_{x \in c_i} d(x,v_i) \]