Time-frequency signal analysis

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Time-Frequency
Signal Analysis Group
www.tfsa.ac.me
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University of Montenegro

- Founded 1974
- Over 20,000 students
- 19 Faculties
- Three cycle education
Faculty of Electrical Engineering

- Founded 1961 as a part of University of Belgrade
- Since 1974 member of the University of Montenegro
- Over 2000 students
- 50 researchers
Research in the area of time-frequency analysis started about 30 years ago, by the end of eighties of the last century. The first papers were presented on conferences few years later, while the first journal paper in this area was published in IEEE Transactions on Signal Processing in February 1993.

More than 300 journal papers have been published on time-frequency signal analysis and other signal processing areas by the members of the Group.

Various signal processing topics have been covered within these papers. The most important ones are in: the fundamental theory of time-frequency analysis, noise influence, higher order time-varying spectra, instantaneous frequency estimation, robust signal processing, time-varying filtering, processing of sparse signals, graph signal processing, vertex-frequency analysis ...
Various applications of time-frequency analysis have been studied, like watermarking, car engine analysis, motion parameters estimation, radar and sonar signal processing. Members of the group are also active in the research related to the compressive sensing algorithms and their relation to the robust analysis and signals with time-varying spectra.

Papers of the members of the Group have been cited more than 10,000 times.

Research of the Group has been supported over the years by many international foundations for the promotion of science.

Time-frequency analysis group of the University of Montenegro has very intensive international collaboration with colleagues from all over the world with frequent mutual visits (almost 300 co-authors).

Web site: www.tfsa.ac.me
The TFSA group is founded and lead by Prof. Ljubiša Stanković

- Full professor at the University of Montenegro
- Member and vice president of the Montenegrin Academy of Sciences and Arts (CANU)
- Fellow IEEE since 2012.
- Member of the International Advisory Board of the Alexander von Humboldt Foundation
- Member of European Academy of Sciences and Arts
- Senior Area Editor, IEEE Transactions on Image Processing
- Member of Editorial Board of Signal Processing since 2014.
Nonstationary signals

- changes spectral content in time
- they appear in many applications, including radar, acoustics, biomedicine, communications, multimedia, seismic, and car industry
- multi-component nonstationary signal is modeled as

\[ x(t) = \sum_{m=1}^{M} A_m(t)e^{j\varphi_m(t)} \]

Time frequency representations

- tools for nonstationary signal analysis
- Spectrogram (short-time Fourier transform)
- Wigner distribution
- S-method
- Choi Williams distribution, Born Jordan distribution, Cohen class . . .
Short time Fourier transform (STFT) is linear, simple, numerically efficient and cross-terms free signal transformation.

\[
\text{STFT}(t, \omega) = \int_{-\infty}^{\infty} x(t + \tau) \omega(\tau) e^{-j\omega \tau} d\tau
\]

\[
\text{STFT}(n, k) = \sum_{m=-N/2}^{N/2-1} \omega(m) x(n + m) e^{-j\frac{2\pi}{N} mk}
\]

Spectral localization over time is obtained using localization window \( \omega(\tau) \) (\( \omega(m) \) in the discrete case).

Spectrogram is an energetic version of the STFT.

\[
\text{SPEC}(t, \omega) = |\text{STFT}(t, \omega)|^2 \\
\text{SPEC}(n, k) = |\text{STFT}(n, k)|^2
\]
Wigner Distribution

- WD is quadratic, highly concentrated, signal energy distribution defined as

$$WD(t, \omega) = \int_{-\infty}^{\infty} w\left(\frac{\tau}{2}\right) w\left(-\frac{\tau}{2}\right) x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right) e^{-j\omega \tau} d\tau$$

$$WD(n, k) = \sum_{m=-N/2}^{N/2-1} w(m) w(-m) x(n + m) x^*(n - m) e^{-j\frac{4\pi}{N} mk}$$

- Undesired cross-terms appears between signal components.
- Time-frequency energy localization is obtained by quadratic signal form, rather than by window $w(\tau)$. 

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Wigner Distribution illustration

\[ x(t) \]

\[ x(t + \tau/2) \]

\[ x(t - \tau/2) \]

\[ x(t + \tau/2)x^*(t - \tau/2) \]

\[ FT\{x(t + \tau/2)x^*(t - \tau/2)\} \]

\[ WD(t, \Omega) \]
Wigner Distribution and STFT

- Spectrogram is cross-terms free but with low concentration in time-frequency plane.
- Wigner distribution is highly concentrated but cross-terms (between signal components) appears.
- Wigner distribution can be calculated from the STFT as

\[
WD(t, \omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \text{STFT}(t, \omega + \theta) \text{STFT}^*(t, \omega - \theta) d\theta
\]

\[
WD(n, k) = \sum_{i=-N/2}^{N/2} \text{STFT}(n, k + i) \text{STFT}^*(n, k - i)
\]
S-method

- S-method can provide cross-terms free highly concentrated time-frequency representation.
- This is achieved by introducing frequency domain window $P(\theta)$ (or $P(i)$) in the WD – STFT relation.

$$SM(t, \omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} P(\theta) \text{STFT}(t, \omega + \theta) \text{STFT}^*(t, \omega - \theta) d \theta$$

- This window restrict integration (summation) limits. The simplest case is rectangular window

$$SM(n, k) = \sum_{i=-L}^{L} \text{STFT}(n, k + i) \text{STFT}^*(n, k - i)$$

$$= \text{SPEC}(n, k) + 2 \sum_{i=1}^{L} \Re \{ \text{STFT}(n, k + i) \text{STFT}^*(n, k - i) \}$$
S-method illustration

Short time Fourier transform

\[ |S_N(0, k)|^2 = SM_0(0, k) \]

STFT

\[ 2\Re[S_N(0, k + 1)S_N^*(0, k - 1)] \]

first correction term

(b)+(c)=(d)

\[ 2\Re[S_N(0, k + 2)S_N^*(0, k - 2)] \]

second correction term

(d)+(e)=(f)

\[ SM_1(0, k) \]

\[ SM_2(0, k) \]
S-method illustration

SM<sub>0</sub>(0, k)

SM<sub>3</sub>(0, k)

SM<sub>8</sub>(0, k)

L = 0

L = 3

L = 8

SM<sub>1</sub>(0, k)

SM<sub>5</sub>(0, k)

SM<sub>31</sub>(0, k) = WD(0, k)

-32 -16 0 16 31

-32 -16 0 16 31

-32 -16 0 16 31

-32 -16 0 16 31

-32 -16 0 16 31

-32 -16 0 16 31
Time frequency representations example

Consider four-component signal with components:

- Constant frequency component
- Linear frequency modulated component
- Two Gaussian components (modulated)

Time-frequency contents of the signal will be analyzed using

- Spectrogram
- Wigner distribution
- Choi-Williams distribution
- S-method
Time-frequency representations example
The S-method is successfully applied to many signal processing problems including:

- Obtaining highly concentrated time-frequency representation of a given signal
- Instantaneous frequency estimation
- Multicomponent signal decomposition
- Focusing of radar images
- Time-varying filtering
- ...

Could the S-method be useful for gravitational waves?
Here we will analyze the data obtained from:

- https://www.kaggle.com/elenacuoco/the-gravitational-waves-discovery-data
- https://www.gw-openscience.org/GW150914data/P150914/fig1-observed-L.txt
- This dataset is txt file with 3441 data samples.
- Prior to the analysis downsampling with factor 16 is applied resulting in signal with 216 samples.

This research has made use of data, software and/or web tools obtained from the LIGO Open Science Center (https://losc.ligo.org), a service of LIGO Laboratory and the LIGO Scientific Collaboration. LIGO is funded by the U.S. National Science Foundation.
Signal in time domain
Signal in frequency domain

Fourier transform (abs)

frequency

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Spectrogram, window length: 128
S-method, L=1
S-method, $L=2$
S-method, L=3

The diagram shows a time-frequency analysis with time on the x-axis and frequency on the y-axis. The S-method with L=3 is used to visualize the signal's frequency content over time, highlighting the dynamic changes in frequency over the specified time range.
S-method, L=4
S-method, $L=5$
Matlab/Octave code

% signal x is zero padded on both sides with M/2 zeros
M = 128; % window length
N = length(x); % signal length
wf = hanning(M); % window function for STFT

S = zeros(M,N-M); % STFT calculation
tau = 0:M-1;
for k = 1:N-M
    y = x(k+tau).*wf;
    S(:,k) = fft(y);
end

L = 5; % S-method calculation
SM = abs(S).^2; % start form Spectrogram
for k = 1:L % add correction terms
    SM(k+1:end-k,:) = SM(k+1:end-k,:) +
    2*real(S(1:end-2*k,:).*conj(S(2*k+1:end,:)));
end
The culmination of more than twenty years of research, this authoritative resource provides you with a practical understanding of time-frequency signal analysis.

The book offers in-depth coverage of critical concepts and principles, along with discussions on key applications in a wide range of signal processing areas, from communications and optics to radar and biomedicine.

Digital Signal Processing

This book is a result of author’s thirty-three years of experience in teaching and research in signal processing. The book will guide you from a review of continuous-time signals and systems, through the world of digital signal processing, up to some of the most advanced theory and techniques in adaptive systems, time-frequency analysis, and sparse signal processing. It provides simple examples and explanations for each, including the most complex transform, method, algorithm or approach presented in the book.